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Analytic model for transient heat exchanger response

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Abstract

The temperature transient response of a single-phase fluid and a wall in a heat exchanger is investigated for when the other constant temperature fluid is subjected to a step change in temperature or when the single-phase fluid is subjected to a step change in mass flow rate. The dynamic behavior of the heat exchanger is approximated by an integral method assuming that the single-phase fluid temperature distribution can be expressed by a combination of the initial and final temperature distributions and a determined time function. The results are validated by comparison against numerical simulations. Excellent agreement is obtained.

Keywords: Heat exchangers; Transients; Integral solution

1. Introduction

When heat exchangers are part of larger industrial processes and systems, transient operation can occur frequently, and the transient response (outlet conditions) of the heat exchanger can affect overall system performance. For example, if the flow from a heat exchanger were feeding a chemical reactor, a time varying output from the heat exchanger would affect the reactor output. Proper design of the process control system requires that the heat exchanger transient performance (outlet conditions) be predicted. Likewise, how heat exchanger tubes respond to temperature transients has an effect on the thermal stresses created. To predict those stresses, the temperature distribution along the length of the tubes must be known as a function of time.

Most of the studies reported in the literature developed the transient response of heat exchangers after a step change of temperature [1–4] or mass flow rate [4–8] by solving the system's governing differential equations. Because of the presence of partial derivatives, these equations either have no exact analytic solution or the analytic solution is extremely complicated and difficult to use (e.g., [4,8]). Although numerical methods are most often used to solve the differential equations (e.g., [1,9]), they have problems with convergence, stiffness, numerical diffusion, and stability. Methods using conventional transforms (e.g., Laplace transforms [3,10]) lead to problems with inversion of the solution from the transformed domain to the original domain of the independent variable(s); numerical inversion of the Laplace transform must be used, which has the same problems as previously mentioned. Reynolds [11] used an integral method to get an implicit solution for a specialized situation of the cooling of a heat exchanger wall and one single-phase fluid; however, the implicit equation is not easily used.

Explicit equations are desired to describe the transient response of a heat exchanger. Lachi et al. [6] used a two-parameter method including a time constant and a time lag to characterize the behavior of a double pipe and one pass shell-and-tube heat exchanger when a sudden change of the flow rate was imposed at one of the two inlets. Abdelghani-Idrissi et al. [7] described the temperature transient response along a counterflow heat exchanger with two single-phase fluids by a first-order response with a time constant when mass flow rate was subjected to a sudden change. Their analytic solutions are explicit, and their agreement with experimental data

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Nomenclature

A	heat transfer area (m^2)
A_x	cross-sectional flow area (m^2)
C_1–C_3	coefficients
D_1–D_3	coefficients
c_p	specific heat (kJ/kg K)
f, f', g, g'	function of time
L	heat exchanger length (m)
m	mass (kg)
$\dot{m}$	mass flow rate (kg/s)
N_1	$(m_{\mathrm{w}}c_{p\mathrm{w}})/(m_{\mathrm{s}}c_{p\mathrm{s}})$
N_2	$(\alpha_{\mathrm{c}}A_{\mathrm{c}})/(\dot{m}c_{p\mathrm{s}})$
N_3	$(\alpha_{\mathrm{s}}A_{\mathrm{s}})/(\dot{m}c_{p\mathrm{s}})$
NTU	number of transfer units
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
t	time (s)
t_{r}	residence time (s)
T	temperature (K)
u	velocity (m/s)
x	position (m)

Greek symbols

α	heat transfer coefficient (W/m^2 K)
λ, ε	interim parameter
γ	$\dot{m}^{\infty}/\dot{m}^{0}$
ρ	density (kg/m^3)

Subscripts

c	constant temperature fluid
s	single-phase fluid
in	inlet
out	outlet
w	wall

Superscripts

0	initial condition
∞	final condition
$*$	dimensionless form

is good. However, two quantities, the time lag in Ref. [6] and the characteristic time constant in Ref. [7], both depend on additional experimental or numerical information.

Shah [12] and Kays and London [13] referred to solutions for transient heat exchanger response in several specific situations. Only the outlet temperature response was given, and the fluid temperature responses within the exchanger and the wall temperature responses were not presented. Some of solutions were obtained numerically, and others were obtained by electromechanical analog experiments. Several solutions were obtained from analysis but were valid only for some limited situations; either no explicit equations were given or no general explicit solutions were presented.

In this paper, an integral method is used to describe the transient behavior of a heat exchanger, which has one fluid with a constant temperature and the other fluid remains single phase. Because flow arrangement becomes irrelevant when one of the fluids in a heat exchanger has a constant temperature, the results in this paper are applicable to counterflow, parallel flow, crossflow, or any other heat exchanger. These types of heat exchangers are widely used (e.g., condensers, evaporators, intercoolers, pre-coolers, and liquid-to-gas heat exchangers). In such heat exchangers, the heat capacity rate of one fluid often is much larger than that of the other fluid, so that the temperature of the fluid with the larger heat capacity rate can be approximated as constant throughout the exchanger. Two cases are considered: (1) when the constant temperature fluid undergoes a step change in temperature, and (2) when the single-phase fluid undergoes a flow rate step change. Compared to previous analyses, this method is more easily understood, the explicit solution is easily used, and no parameters depend on additional experimental or numerical information.

The present analysis provides the time varying fluid and tube wall temperatures over the whole length of a heat exchanger. These results can be used, for example, in the design and operation of the control system of a larger system in which the heat exchanger is only one part, in a heat exchanger mechanical design if the transient thermal stresses in the tubes are required, and in prediction of the transient overall heat exchanger performance (e.g., time required to change from one steady-state operating condition to another).

2. Governing equations

Consider the heat exchanger shown in Fig. 1. One fluid remains at a constant temperature (e.g., a boiling or condensing process); another fluid remains a single-phase fluid, and its inlet temperature remains constant at all times. Note that Shah [12] studied transient behavior for such heat exchangers by summarizing several authors' methods. Rizika [14] obtained an exact solution, but it was valid only for a time period less than or equal to the residence time of the single-phase flow and only for a liquid flow. London et al. [15] presented analytical solutions for two limiting cases, for when the ratio of the

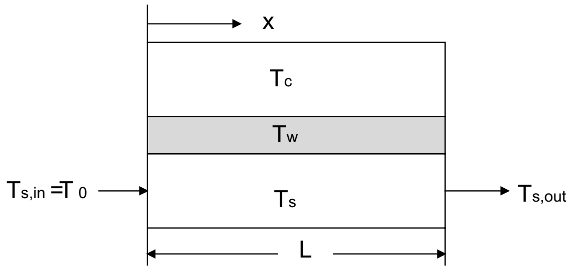

Fig. 1. Schematic of two-fluid heat exchanger.

thermal resistance on the two sides of the heat exchanger was equal to 0 or ∞; other solutions were obtained numerically or by electromechanical analog experiments. Thus, there are no general explicit solutions for a step change in the temperature of constant temperature fluid, and there are no solutions for a step change in the mass flow rate of the single-phase fluid.

Therefore, in the present paper, two transient heat exchanger situations are considered. The first one deals with a temperature step change in the constant temperature fluid, and the second one deals with a step change in the single-phase fluid mass flow rate. The problem is to find the transient single-phase fluid and wall temperature distributions along the whole pipe after the step change.

For this analytic model, the following assumptions are made: the temperatures of the single-phase fluid and the wall are only functions of time t and position x; there are no thermal energy sources within the single-phase fluid or the wall; longitudinal and transverse heat conduction within the wall and the fluid are neglected; the convective heat transfer coefficient on each side of the heat exchanger and the thermal properties of the fluid and the wall are constant. Based on the above assumptions, energy balances on a differential element in the exchanger wall and fluid, respectively, are:

$$m_w c_{pw} \frac{\partial T_w}{\partial t} + \alpha_c A_c (T_w - T_c) + \alpha_s A_s (T_w - T_s) = 0 \quad (1)$$

$$m_s c_{ps} \frac{\partial T_s}{\partial t} + \dot{m} c_{ps} L \frac{\partial T_s}{\partial x} = \alpha_s A_s (T_w - T_s) \quad (2)$$

Eqs. (1) and (2) are rewritten using the following dimensionless variables and parameters:

$$T^* = \frac{T - T_{s,in}}{T_c^\infty - T_{s,in}}, \quad x^* = \frac{x}{L}, \quad t^* = \frac{t}{t_r},$$
$$N_1 = \frac{m_w c_{pw}}{m_s c_{ps}}, \quad N_2 = \frac{\alpha_c A_c}{\dot{m} c_{ps}} \quad \text{and} \quad N_3 = \frac{\alpha_s A_s}{\dot{m} c_{ps}}$$

where t_r is the residence time of single-phase fluid in the heat exchanger, $t_r = L/u = L\rho_s A_x/\dot{m} = m_s/\dot{m}$, T_c and $T_{s,in}$ are both constant. Note that N_1 is the ratio of the wall thermal capacitance to the single-phase fluid thermal capacitance, $N_3 = \mathrm{NTU_s}$ is the number of transfer units based on the single-phase side, and $N_2 = (\alpha_c A_c/\alpha_s A_s)\mathrm{NTU_s}$ is the $\mathrm{NTU_s}$ multiplied by the ratio of the thermal resistance ratio. Incorporating these dimensionless quantities, the non-dimensional forms of Eqs. (1) and (2) are obtained:

$$N_1 \frac{\partial T_w^*}{\partial t^*} + N_2 (T_w^* - 1) + N_3 (T_w^* - T_s^*) = 0 \quad (3)$$

$$\frac{\partial T_s^*}{\partial t^*} + \frac{\partial T_s^*}{\partial x^*} = N_3 (T_w^* - T_s^*) \quad (4)$$

3. Integral method

Integral methods [11] have been used in the analysis of many situations but have not been used in recent years because of the reliance on numerical analyses. However, the approach is still useful. Integral methods apply fundamental concepts and phenomenological relations to an entire system, rather than just to a differential element. Thus, in the present situation, we integrated the governing partial differential equations, Eqs. (3) and (4), over the length of the heat exchanger:

$$N_1 \frac{\partial \left(\int_0^1 T_w^* \, dx^* \right)}{\partial t^*} + N_2 \left(\int_0^1 T_w^* \, dx^* - 1 \right) + N_3 \left(\int_0^1 T_w^* \, dx^* - \int_0^1 T_s^* \, dx^* \right) = 0 \quad (5)$$

$$\frac{\partial \left(\int_0^1 T_s^* \, dx^* \right)}{\partial t^*} + [T_s^*(x^* = 1) - T_s^*(x^* = 0)] = N_3 \left(\int_0^1 T_w^* \, dx^* - \int_0^1 T_s^* \, dx^* \right) \quad (6)$$

In order to evaluate the integrals appearing in Eqs. (5) and (6), we have to assume the forms of the spatial distributions of both the wall and single-phase fluid temperatures. We assume that both temperature distributions have the following forms:

$$T_s^* = T_s^{*0} + (T_s^{*\infty} - T_s^{*0}) f(t^*) \quad (7)$$

$$T_w^* = T_w^{*0} + (T_w^{*\infty} - T_w^{*0}) g(t^*) \quad (8)$$

This assumption uses the known initial and final temperature distributions as was done in Refs. [6,7], which use first-order systems to approximate the transient temperatures for both the fluids and wall(s). In our model, we do not give the specific form of $f(t^*)$ and $g(t^*)$, which are left to be determined.

The assumed behavior of Eqs. (7) and (8) must satisfy three conditions: at $t^* = 0$, $T_s^* = T_s^{*0}$, $T_w^* = T_w^{*0}$; at $t^* \to \infty$, $T_s^* = T_s^{*\infty}$, $T_w^* = T_w^{*\infty}$ and at $x^* = 0$, $T_s^* = 0$, which are the initial, final, and inlet boundary conditions, respectively. In Eqs. (7) and (8), $f(t^*)$ and $g(t^*)$ are only time functions, so we can reduce our system of partial differential equations to two first-order ordinary differential equations, which are more easily solved. Below, two cases are given to explain this method.

3.1. Case 1

At time zero-minus, the whole system is at steady state. At time zero, the constant temperature fluid undergoes an instantaneous step change in temperature, from T_c^0 to T_c^∞. The single-phase fluid inlet temperature is maintained constant at $T_{s,in}$. Thus, the initial condition for the single-phase fluid and heat exchanger wall are readily obtained from Eqs. (3) and (4) because the transient terms disappear. Thus,

$$T_s^{*0} = T_c^{*0}(1 - e^{-NTUx^*}) \tag{9}$$

$$T_w^{*0} = T_c^{*0}\left(1 - \frac{NTU}{N_2}e^{-NTUx^*}\right) \tag{10}$$

where

$$T_c^{*0} = (T_c^0 - T_{s,in})/(T_c^\infty - T_{s,in}) \tag{11}$$

where NTU is the number of transfer units:

$$NTU = \frac{\alpha_c A_c \alpha_s A_s}{\dot{m}c_{ps}(\alpha_c A_c + \alpha_s A_s)} = \frac{N_2 N_3}{N_2 + N_3} \tag{12}$$

(Note that wall thermal resistance can be incorporated approximately into the solution by combining the wall resistance with the convective resistance on either side of the heat exchanger.)

The final steady-state temperature distributions, $T_s^{*\infty}$ and $T_w^{*\infty}$, are obtained by solving Eqs. (3) and (4) at the final steady state:

$$T_s^{*\infty} = 1 - e^{-NTUx^*} \tag{13}$$

$$T_w^{*\infty} = 1 - \frac{NTU}{N_2}e^{-NTUx^*} \tag{14}$$

Substituting the initial and final steady-state conditions (Eqs. (9), (10), (13) and (14)) into Eqs. (7) and (8), and rearranging the equations, the single-phase fluid and wall temperature distributions are obtained:

$$T_s^* = (1 - e^{-NTUx^*})\{f(t^*) + T_c^{*0}[1 - f(t^*)]\} \tag{15}$$

$$T_w^* = \left(1 - \frac{NTU}{N_2}e^{-NTUx^*}\right)\{g(t^*) + T_c^{*0}[1 - g(t^*)]\} \tag{16}$$

Since T_c^{*0} is a constant, $\{f(t^*) + T_c^{*0}[1 - f(t^*)]\}$ and $\{g(t^*) + T_c^{*0}[1 - g(t^*)]\}$ are only time functions; thus, we can describe these two terms as $f'(t^*)$ and $g'(t^*)$ to simplify Eqs. (15) and (16):

$$T_s^* = (1 - e^{-NTUx^*})f'(t^*) \tag{17}$$

$$T_w^* = \left(1 - \frac{NTU}{N_2}e^{-NTUx^*}\right)g'(t^*) \tag{18}$$

Note that when $f'(t^* = 0) = g'(t^* = 0) = T_c^{*0}$ and $f'(t^* = \infty) = g'(t^* = \infty) = 1$, the initial and final steady-state temperature distributions, respectively, are satisfied. When $x^* = 0$, the inlet boundary condition is satisfied.

Substituting Eqs. (17) and (18) into Eqs. (5) and (6), carrying out the integration, and rearranging them, two first-order ordinary differential equations are obtained:

$$\frac{df'(t^*)}{dt^*} = C_1 f'(t^*) + C_2 g'(t^*) \tag{19}$$

$$\frac{dg'(t^*)}{dt^*} = D_1 f'(t^*) + D_2 g'(t^*) + D_3 \tag{20}$$

where

$$C_1 = -C_2 = -\frac{N_3 NTU(N_2 - \varepsilon)}{N_2(NTU - \varepsilon)} \tag{21}$$

$$D_1 = \frac{(N_3 + N_2)(NTU - \varepsilon)}{N_1(N_2 - \varepsilon)} \tag{22}$$

$$D_2 = -\frac{N_2 + N_3}{N_1} \tag{23}$$

$$D_3 = \frac{N_2^2}{N_1(N_2 - \varepsilon)} \tag{24}$$

where $\varepsilon = 1 - e^{-NTU}$ is the steady-state heat exchanger effectiveness for a heat exchanger with a heat capacity ratio of zero.

These two ordinary differential equations can be solved, subject to the initial conditions, $f'(t^* = 0) = g'(t^* = 0) = T_c^{*0}$, and $(C_1 - D_2)^2 + 4D_1C_2 > 0$ for all situations. Thus, $f'(t^*)$ and $g'(t^*)$ are obtained:

$$f'(t^*) = \frac{\lambda_2 e^{\lambda_1 t^*} - \lambda_1 e^{\lambda_2 t^*}}{\lambda_1 - \lambda_2}[1 - T_c^{*0}] + 1 \tag{25}$$

$$g'(t^*) = 1 - \frac{(1 - T_c^{*0})[\lambda_2(\lambda_1 - C_1)e^{\lambda_1 t^*} - \lambda_1(\lambda_2 - C_1)e^{\lambda_2 t^*}]}{C_1(\lambda_1 - \lambda_2)} \tag{26}$$

where

$$\lambda_{1,2} = \frac{(C_1 + D_2) \pm \sqrt{(C_1 - D_2)^2 + 4D_1C_2}}{2} \tag{27}$$

Thus, T_s^* (Eq. (17)) and T_w^* (Eq. (18)) are now known functions of space and time. Note that $\lambda_1 \neq \lambda_2$ for all conditions.

When $T_c^{*0} = 0$, the initial condition is a uniform temperature distribution everywhere because $T_s^{*0} = T_w^{*0} = 0$, and the initial mass flow rate plays no role in the solution. Hence, this is the classic "start up" situation. That is, at time zero, a step change in temperature is imposed to the constant temperature fluid, and the single-phase fluid undergoes an instantaneous step change in mass flow rate.

3.1.1. Evaluation of solution

For comparison to the approximate analytic solution given in the previous section and in the following sections, the complete differential equations, Eqs. (3) and (4), were solved numerically using the Modelica language and a dynamic simulator, Dymola. Modelica is an object-oriented modeling language, which can reuse physical models, support a hierarchical structure, and offer the feature of easy code maintenance by combining properties from basic library components [16]. For evaluation of the models, the simulation environment Dymola (Dynamic Modeling Laboratory) is used [17]. The governing equations were discretized axially and temporally. Grid independence in both space and time were obtained by successive halving of the increments. The results from the present numerical models were validated against the numerical results of Romie [1] and Roetzel and Xuan [3] for both a parallel flow heat exchanger and a counterflow heat exchanger, and the agreement was excellent. Based on this benchmarking exercise, we then used our numerical simulations to validate our approximate analytic model.

As shown previously, the three coefficients in the governing equations affect the transient response. Because the single-phase fluid may be a liquid or a gas, N_1 can vary widely. In most of heat exchangers, NTU is less than about 6 which limits the variations in N_2 and N_3. Many combinations of N_1, N_2, and N_3 were studied. Because of space limitations, we show only typical results. Figs. 2–5 compare the results from the analytic model and the numerical simulation for $T_c^{*0} = 0$. Figs. 2–4 give the outlet temperature response for the single-phase fluid and the wall for a range of N_1, at three different values of NTU. Generally, good agreement is obtained. Fig. 5 shows the outlet temperature response for the single-phase fluid and wall for a range of N_2 and N_3 when $N_1 = 1$. Because the present solution is obtained by solving the governing *integral* equations, Eqs. (5) and (6), the governing *differential* equations, Eqs. (3) and (4), are satisfied only on the average and may be only approximately satisfied at each point. Thus, for some combinations of N_1, N_2 and N_3, the agreement between the analytic and numeric solution degrades. However, studies show that for most situations, the differences between the analytic model and numeric solutions are within a couple of percent, and even in the poorer situations, the maximum differences are less than 15–20%, but the trends and steady-state times are still modeled well.

Figs. 6 and 7 compare the results from the analytic model and the numerical simulation for $T_c^{*0} = 0.4$. Fig. 6 shows the temperature distributions along the heat exchanger for the single-phase fluid and the wall at different times when $N_2 = N_3 = 1(\mathrm{NTU} = 0.5)$ and $N_1 = 300$. The agreements are quite good not only at the exit, but also for every location along the heat exchanger. Fig. 7 shows the temperature distributions along the heat exchanger for the single-phase fluid at different times for two different values of NTU and $N_1 = 300$. The agreement is quite good over the whole heat exchanger.

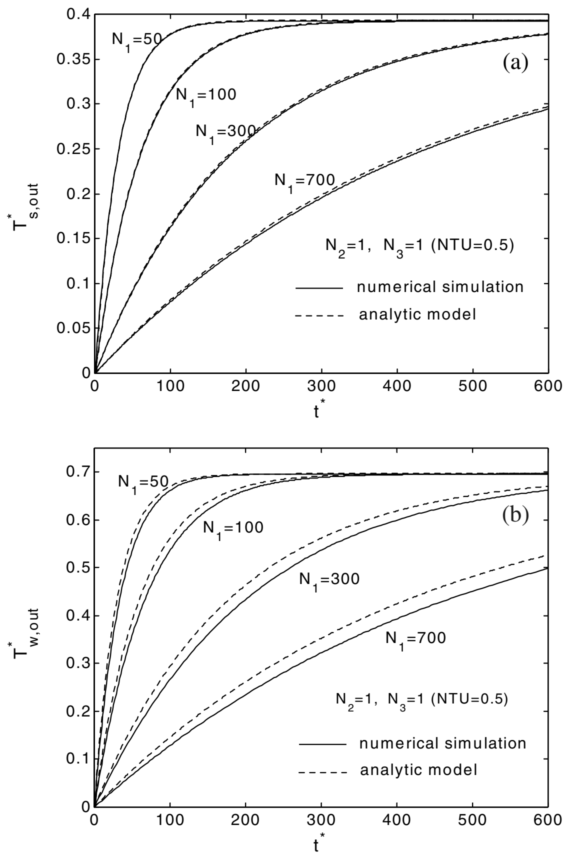

Fig. 2. Outlet fluid and wall temperature transients with $T_c^{*0} = 0$ and $\mathrm{NTU} = 0.5$ (Case 1).

The accuracy of the integral method used in this paper depends upon the choice of the assumptions for the spatial distributions of the wall and single-phase fluid temperatures. From the above studies, we can see that the agreement between the analytic model and the numerical simulation generally is quite good. Thus, we can say that the assumptions we made in Eqs. (7) and (8) are reasonable and suitable.

3.2. Case 2

A step change in the single-phase fluid flow rate is imposed on a heat exchanger operating at a steady state.

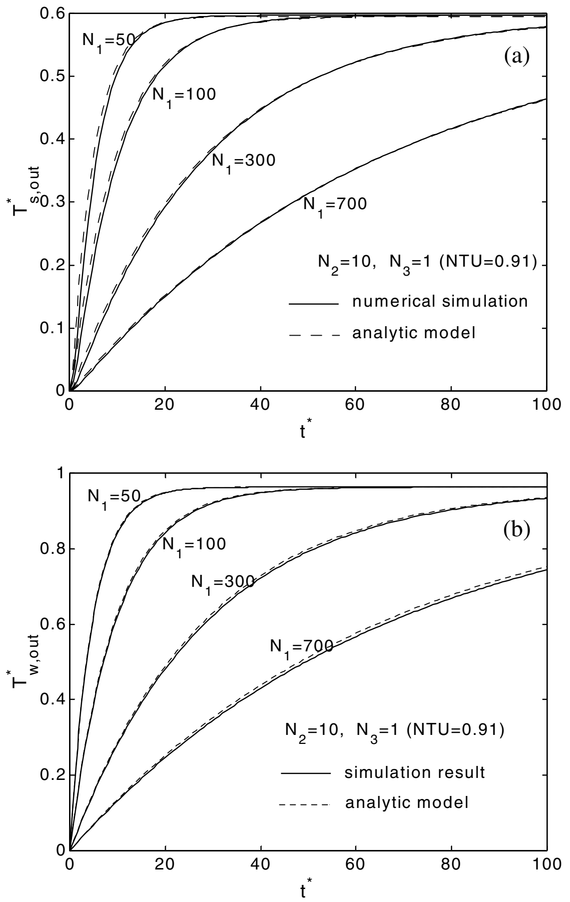

Fig. 3. Outlet fluid and wall temperature transients with $T_c^{*0} = 0$ and NTU = 0.91 (Case 1).

Before the step change, the single-phase inlet mass flow rate is $\dot{m}^0$. Eqs. (3) and (4) can be written as

$$N_1 \frac{\partial T_w^*}{\partial t^*} + N_2^0 (T_w^* - 1) + N_3^0 (T_w^* - T_s^*) = 0 \tag{28}$$

$$\frac{\partial T_s^*}{\partial t^*} + \frac{\partial T_s^*}{\partial x^*} = N_3^0 (T_w^* - T_s^*) \tag{29}$$

where $N_2^0 = (\alpha_c A_c)/(\dot{m}^0 c_{ps})$ and $N_3^0 = (\alpha_s^0 A_s)/(\dot{m}^0 c_{ps})$. At time zero-minus, the whole system is at a steady state. Thus, the initial conditions for single-phase fluid and heat exchanger wall are obtained by solving Eqs. (28) and (29):

$$T_s^{*0} = (1 - e^{-\mathrm{NTU}^0 x^*}) \tag{30}$$

$$T_w^{*0} = \left(1 - \frac{\mathrm{NTU}^0}{N_2^0} e^{-\mathrm{NTU}^0 x^*}\right) \tag{31}$$

where $\mathrm{NTU}^0 = (N_2^0 N_3^0)/(N_2^0 + N_3^0)$.

At time zero, the single-phase fluid undergoes an instantaneous step change in mass flow rate, from $\dot{m}^0$ to

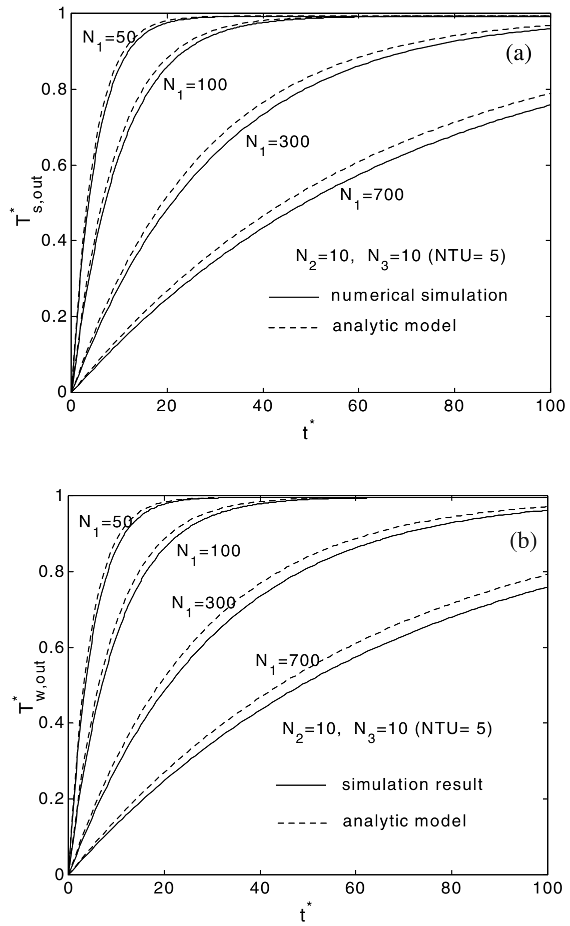

Fig. 4. Outlet fluid and wall temperature transients with $T_c^{*0} = 0$ and NTU = 5 (Case 1).

$\dot{m}^\infty$. The constant temperature fluid temperature and single-phase fluid inlet temperature are maintained constant at T_c^∞ and $T_{s,in}$, respectively, and Eqs. (3) and (4) can be written as:

$$N_1 \frac{\partial T_w^*}{\partial t^*} + N_2^\infty (T_w^* - 1) + N_3^\infty (T_w^* - T_s^*) = 0 \tag{32}$$

$$\frac{\partial T_s^*}{\partial t^*} + \frac{\partial T_s^*}{\partial x^*} = N_3^\infty (T_w^* - T_s^*) \tag{33}$$

where $N_2^\infty = (\alpha_c A_c)/(\dot{m}^\infty c_{ps})$, $N_3^\infty = (\alpha_s^\infty A_s)/(\dot{m}^\infty c_{ps})$. The final steady-state temperature distributions, $T_s^{*\infty}$ and $T_w^{*\infty}$, are readily obtained from the above equations. Thus,

$$T_s^{*\infty} = 1 - e^{-\mathrm{NTU}^\infty x^*} \tag{34}$$

$$T_w^{*\infty} = 1 - \frac{\mathrm{NTU}^\infty}{N_2^\infty} e^{-\mathrm{NTU}^\infty x^*} \tag{35}$$

where $\mathrm{NTU}^\infty = (N_2^\infty N_3^\infty)/(N_2^\infty + N_3^\infty)$.

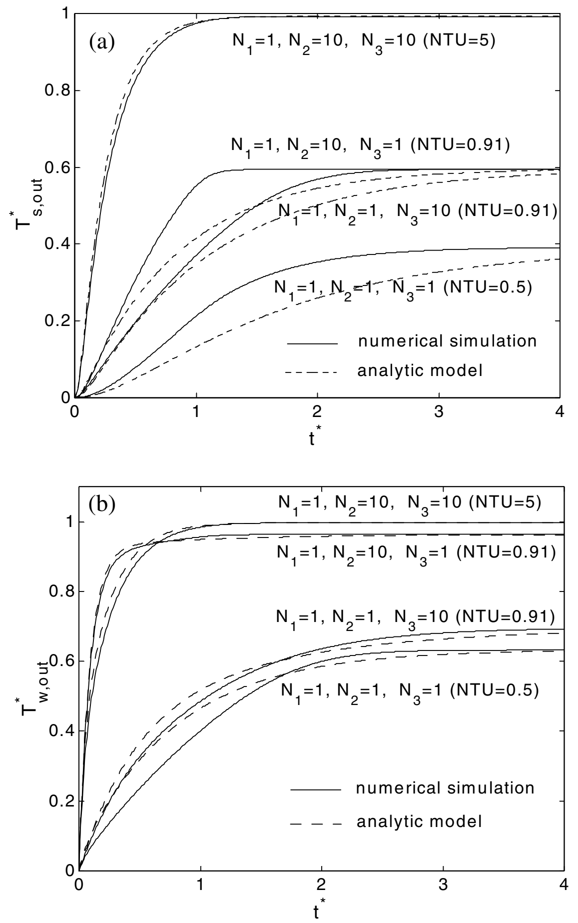

Fig. 5. Outlet fluid and wall temperature transients with $T_c^{*0} = 0$, $N_1 = 1.0$ and various NTU (Case 1).

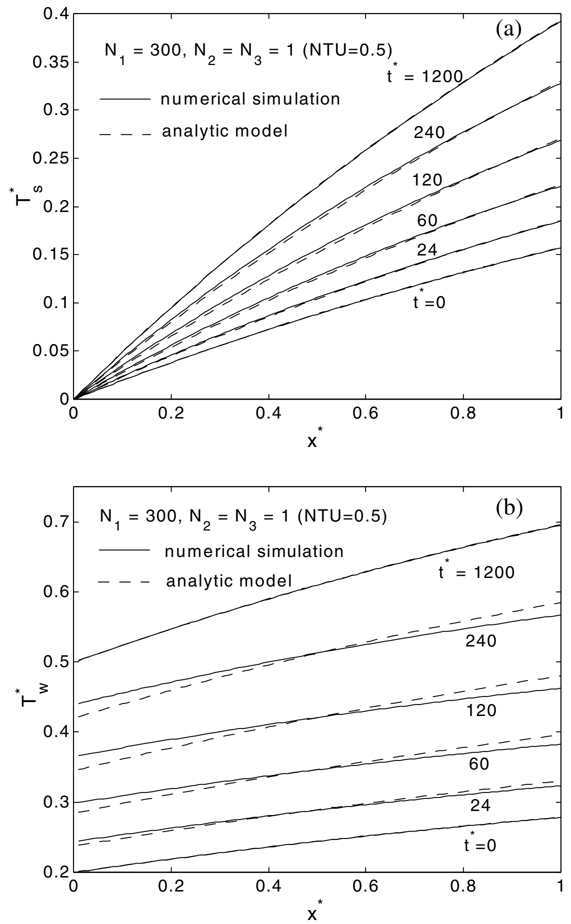

Fig. 6. Transient fluid and wall temperature distributions along the heat exchanger with $T_c^{*0} = 0.4$ and NTU = 0.5 (Case 1).

The heat transfer coefficient on the constant fluid side, α_c, is assumed to remain constant before and after the mass flow rate step change on the single-phase side. However, the heat transfer coefficient for the single-phase flow does change with the step change in mass flow rate. Thus, we must consider how the three parameters N_1, N_2 and N_3 are affected. Because neither flow rate nor heat transfer coefficient are used in N_1, N_1 does not change. For fully developed laminar flow, in a constant shape duct, Nu = constant, so $\alpha_s^\infty = \alpha_s^0$ and

$$\frac{N_3^\infty}{N_3^0} = \frac{N_2^\infty}{N_2^0} = \frac{\dot{m}^0}{\dot{m}^\infty} = \frac{1}{\gamma} \tag{36}$$

where $\gamma = \dot{m}^\infty/\dot{m}^0$. For turbulent flow, using, for example, the Colburn equation:

$$Nu = 0.023Re^{0.8}Pr^{1/3} \tag{37}$$

Thus,

$$\frac{\alpha_s^\infty}{\alpha_s^0} = \left(\frac{Re^\infty}{Re^0}\right)^{0.8} = \left(\frac{u^\infty}{u^0}\right)^{0.8} = \left(\frac{\dot{m}^\infty}{\dot{m}^0}\right)^{0.8} = \gamma^{0.8} \tag{38}$$

$$\left.\begin{aligned} &\frac{N_3^\infty}{N_3^0} = \frac{\alpha_s^\infty \dot{m}^0}{\alpha_s^0 \dot{m}^\infty} = \left(\frac{\dot{m}^0}{\dot{m}^\infty}\right)^{0.2} = \left(\frac{1}{\gamma}\right)^{0.2} \\ &\text{and} \\ &\frac{N_2^\infty}{N_2^0} = \frac{\dot{m}^0}{\dot{m}^\infty} = \frac{1}{\gamma} \end{aligned}\right\} \tag{39}$$

The relationships given in Eqs. (36) and (39) are used only to find the values of N_2^∞ and N_3^∞, but are not needed to solve the governing Eqs. (34) and (35).

Using the initial and final steady-state conditions, we can recast Eqs. (7) and (8) for the problem as:

$$T_s^* = (1 - e^{-\mathrm{NTU}^0 x^*}) + (e^{-\mathrm{NTU}^0 x^*} - e^{-\mathrm{NTU}^\infty x^*})f(t^*) \tag{40}$$

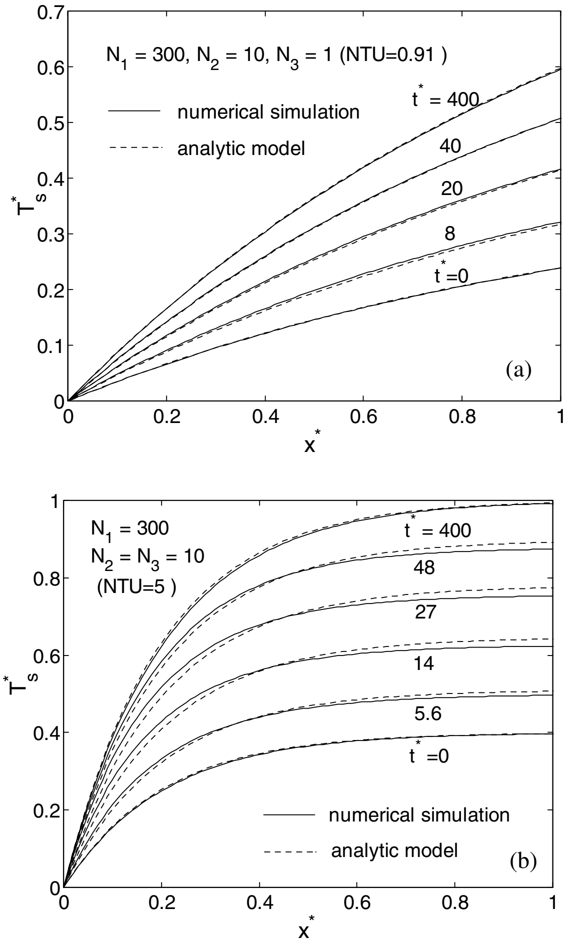

Fig. 7. Transient fluid temperature distributions along the heat exchanger with $T_c^{*0} = 0.4$ and NTU $= 0.91$ and NTU $= 5$ (Case 1).

$$T_w^* = 1 - \frac{\mathrm{NTU}^0}{N_2^0} e^{-\mathrm{NTU}^0 x^*} + \left(\frac{\mathrm{NTU}^0}{N_2^0} e^{-\mathrm{NTU}^0 x^*} - \frac{\mathrm{NTU}^\infty}{N_2^\infty} e^{-\mathrm{NTU}^\infty x^*} \right) g(t^*) \quad (41)$$

Let $f(t^* = 0) = g(t^* = 0) = 0$ and $f(t^* = \infty) = g(t^* = \infty) = 1$; thus, the initial and final steady-state temperature distributions, respectively, are satisfied, and when $x^* = 0$, the inlet boundary condition also is satisfied.

Using the final values of N_2^∞ and N_3^∞, the integral equations become

$$N_1 \frac{\partial (\int_0^1 T_w^* \, dx^*)}{\partial t^*} + N_2^\infty \left(\int_0^1 T_w^* \, dx^* - 1 \right) + N_3^\infty \left(\int_0^1 T_w^* \, dx^* - \int_0^1 T_s^* \, dx^* \right) = 0 \quad (42)$$

$$\frac{\partial \left(\int_0^1 T_s^* \, dx^* \right)}{\partial t^*} + [T_s^*(x^* = 1) - T_s^*(x^* = 0)] = N_3^\infty \left(\int_0^1 T_w^* \, dx^* - \int_0^1 T_s^* \, dx^* \right) \quad (43)$$

Substituting the assumed single-phase and wall temperature distributions (Eqs. (40) and (41)) into Eqs. (42) and (43), carrying out the integration, and rearranging them, two first-order ordinary differential equations are obtained:

$$\frac{df(t^*)}{dt^*} = C_1 f(t^*) + C_2 g(t^*) + C_3 \quad (44)$$

$$\frac{dg(t^*)}{dt^*} = D_1 f(t^*) + D_2 g(t^*) + D_3 \quad (45)$$

where

$$C_1 = \frac{N_3^\infty \varepsilon^\infty / N_2^\infty - (N_3^\infty / \mathrm{NTU}^0 - 1) \varepsilon^0}{\varepsilon^0 / \mathrm{NTU}^0 - \varepsilon^\infty / \mathrm{NTU}^\infty} \quad (46)$$

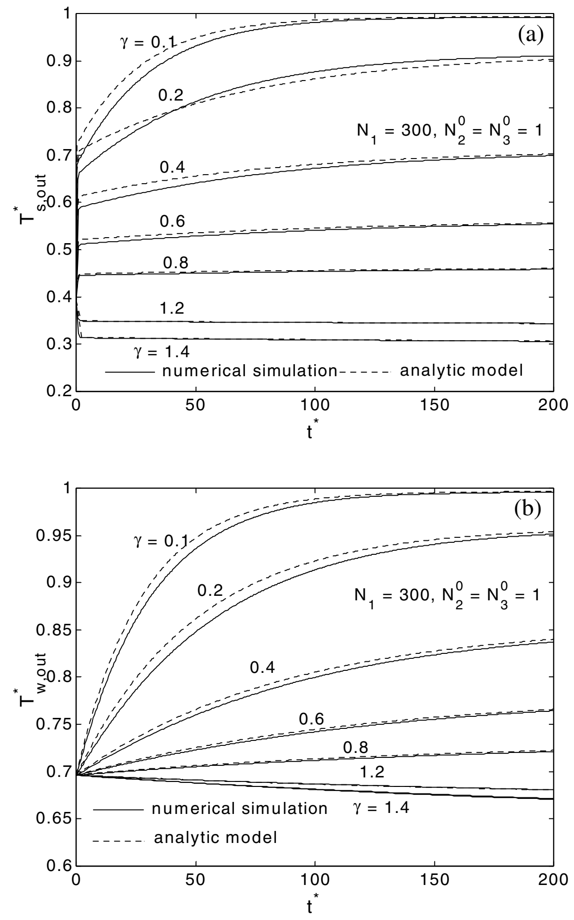

Fig. 8. Outlet fluid and wall temperature transients with $\mathrm{NTU}^0 = 0.5$ and various γ (Case 2).

$$C_2 = \frac{N_3^{\infty}\varepsilon^0/N_2^0 - N_3^{\infty}\varepsilon^{\infty}/N_2^{\infty}}{\varepsilon^0/\mathrm{NTU}^0 - \varepsilon^{\infty}/\mathrm{NTU}^{\infty}} \tag{47}$$

$$C_3 = \frac{\varepsilon^0(N_3^{\infty}/N_3^0 - 1)}{\varepsilon^0/\mathrm{NTU}^0 - \varepsilon^{\infty}/\mathrm{NTU}^{\infty}} \tag{48}$$

$$D_1 = \frac{N_3^{\infty}(\varepsilon^0/\mathrm{NTU}^0 - \varepsilon^{\infty}/\mathrm{NTU}^{\infty})}{N_1(\varepsilon^0/N_2^0 - \varepsilon^{\infty}/N_2^{\infty})} \tag{49}$$

$$D_2 = -\frac{(N_2^{\infty} + N_3^{\infty})}{N_1} \tag{50}$$

$$D_3 = -\frac{\varepsilon^0 N_3^{\infty}[1/\mathrm{NTU}^0 - N_2^{\infty}/(N_2^0\mathrm{NTU}^{\infty})]}{N_1(\varepsilon^0/N_2^0 - \varepsilon^{\infty}/N_2^{\infty})} \tag{51}$$

where $\varepsilon^0 = 1 - \mathrm{e}^{-\mathrm{NTU}^0}$ and $\varepsilon^{\infty} = 1 - \mathrm{e}^{-\mathrm{NTU}^{\infty}}$.

These two ordinary differential equations can be solved, subject to the initial conditions, $f(t^* = 0) = g(t^* = 0) = 0$. Thus, $f(t^*)$ and $g(t^*)$ are obtained

$$f(t^*) = 1 - \frac{(\lambda_2 + C_3)\mathrm{e}^{\lambda_1 t^*} - (\lambda_1 + C_3)\mathrm{e}^{\lambda_2 t^*}}{(\lambda_2 - \lambda_1)} \tag{52}$$

$$g(t^*) = 1 - \frac{(\lambda_2 + C_3)(\lambda_1 - C_1)\mathrm{e}^{\lambda_1 t^*} - (\lambda_1 + C_3)(\lambda_2 - C_1)\mathrm{e}^{\lambda_2 t^*}}{C_2(\lambda_2 - \lambda_1)} \tag{53}$$

where $\lambda_{1,2}$ has the same definition as before (Eq. (27)).

Substituting Eqs. (52) and (53) into Eqs. (40) and (41), T_s^* and T_w^* are obtained.

3.2.1. Evaluation of solution

Figs. 8 and 9 give the effect of different γ on the outlet temperature distribution for the single-phase fluid and for the wall. When the mass flow rate of the single-phase fluid undergoes a step change, the temperature response for the single-phase fluid is very fast. As discussed above, using the integral method, the governing equations are satisfied on the average, but may be only approximately satisfied at each point in the system [11]. Thus, at very short times, relatively large differences exist between the analytic model and numeric solutions for single-phase fluid. The agreement with the wall

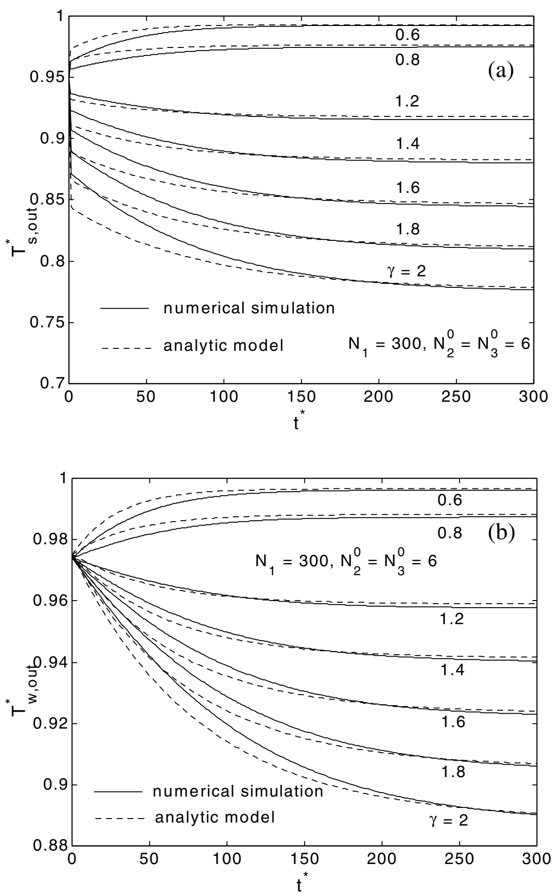

Fig. 9. Outlet fluid and wall temperature transients with $\mathrm{NTU}^0 = 3$ and various γ (Case 2).

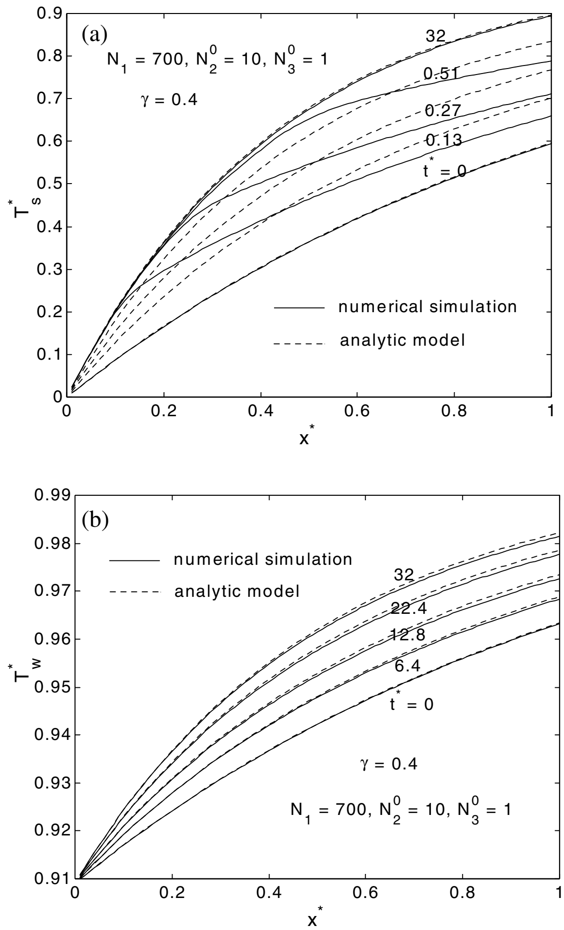

Fig. 10. Transient fluid and wall temperature distributions along the heat exchanger with $\mathrm{NTU}^0 = 0.91$ and $\gamma = 0.4$ (Case 2).

temperature is much better because it is affected more slowly compared with the single-phase fluid. Fig. 10 shows the temperature distributions within the heat exchanger for the single-phase fluid and wall at different times when $N_2^0 = 10$, $N_3^0 = 1$ (NTU $= 0.91$), $N_1 = 700$ and $\gamma = 0.4$. The agreement is good, especially for wall temperature as explained above.

4. Concluding remarks

The transient behavior of a heat exchanger in which one fluid is single phase and the other has a constant temperature has been investigated for the situation when there is a step change in temperature and/or mass flow rate. The transient temperature distributions in the single-phase fluid and wall were obtained using an integral method by assuming that the single-phase fluid and wall temperatures can be expressed by a combination of known initial and final temperature distributions and a time function determined in the analysis. For two cases, explicit analytic solutions were obtained and validated by comparison against numerical simulation results over a wide range of operating conditions. The results show that this model covers most practical cases with very good accuracy.

Because flow arrangement becomes irrelevant when one of the fluids in a heat exchanger has a constant temperature, the results in this paper are applicable to counterflow, parallel flow, cross-flow, or any other heat exchanger. The present analytic solutions can provide good predictions for transient performance of such heat exchangers, and, thus, provide good guidance for proper control of heat exchangers. The solution method employed in this paper may be extendable to heat exchangers with two single-phase fluids. We are investigating that possibility.

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